Scheme-based Definition and Conjecture Synthesis for Inductive Theories

Omar Montano, Roy McCasland, Lucas Dixon, Alan Bundy

School of Informatics,
University of Edinburgh, Informatics Forum, 10 Crichton Street,
Edinburgh EH8 9AB, Scotland

1 Introduction

Human mathematical discovery processes include the invention of definitions, conjectures, theorems, examples, problems and algorithms for solving these problems. Automating these processes is an exciting area for research which is now recognised by the automated reasoning community [5, 3, 7, 6, 4]. What is central to automated discovery programs is the generation of conjectures and definitions. Here we present a theory for mathematical theory exploration. The theory is grounded in a concept called scheme [2]. A scheme is a higher-order formula intended to generate new definitions (definitional schemes) of the underlying theory and conjectures (propositional schemes) about them. The invention process is carried out through the instantiation of free variables within the scheme with closed terms of the theory.

Instantiated schemes are then normalized w.r.t. a normalizing term rewrite system (see [1]) $\mathcal{R}$ built from the defining equations of definitions and lemmata discovered during the exploration of the theory. Equivalent instantiations (modulo $\mathcal{R}$) are identified by structural comparison of terms (up to variable renaming). The rewrite system $\mathcal{R}$ is not only used to reduce redundancies, inherent in most theory formation systems, but also to help with the proof obligations during the exploration of the theory. To asses the quality of the theorems discovered we perform a precision/recall analysis using Isabelle's theorem library as reference. We argue that the quality of a definition can be estimated by the number of theorems involving that definition at the end of the exploration process.

References

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